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A Cooperative Learning Programme to Enhance Mathematical Problem Solving Performance among Secondary Three Students

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Abstract: This article described a programme that aimed to enhance students' mathematical problem-solving performance through cooperative learning. In the whole-class study, the researcher planned and implemented the cooperative problem-solving programme for 40 Secondary Three female students. Results using pre and post tests showed that the class scored better in problem-solving performance after the programme. The researcher further selected a group of four students was further selected to study individual student behaviours by audio-taping their discussions in order to identify metacognitive and affective behaviours during cooperative learning. The results revealed that the four students exhibited metacognitive behaviours such as asking for clarification, giving a suggestion, evaluating solutions; and affective behaviours such as persisting in the task, praising and encouraging when they solved problems in a group.

Key words: Cooperative learning; Problem solving; Learning strategies; Secondary students

Introduction

Mathematical problem solving was advocated in the "Agenda for Action" by the National Council of Teachers of Mathematics (NCTM) in the United States in 1980 to be the focus of school mathematics. The NCTM "Curriculum and Evaluation Standards for School Mathematics" (1989) placed 'Mathematics as Problem Solving' to be an integral component of mathematics curriculum. Moving in the same direction, the Singapore mathematics syllabus for schools was revised in the late 1980s and moved from the focus on arithmetic and computational skills towards problem solving (Ministry of Education, 1990, 2000, 2001). The problem-solving framework described the attainment of problem-solving ability as dependent upon five inter-related components: concepts, skills, processes, attitudes, and metacognition. In the latest revision of the curriculum framework, the processes component is expanded to include mathematical reasoning, communication and connections, in addition to thinking skills and problem-solving heuristics (Ministry of Education, 2007).

Currently, most mathematics classrooms in Singapore are engaged in the traditional approach of whole class expository instruction followed by numerous individual drill and practice activities (Chang, Kaur, Koay, & Lee, 2001). Hence there is a need to explore other instructional strategies that encourage active student interaction in the classroom, and provide opportunities for students to express their mathematical ideas, sharpen their reasoning skills and improve their communication skills during problem solving.

Davidson (1990) believed that small-group cooperative learning could be used to foster effective mathematical communication, problem solving, logical reasoning and the making of mathematical connections. By putting the students into cooperating groups, they knew that they were not handling the problems alone and they would be helping one another cope with the unfamiliar problem situations. Studies by Gillies and Boyle (2005) suggest that teachers' behaviours during cooperative learning play an important role too. Their studies revealed that children model many of the verbal interactions they have seen demonstrated by their teacher and with each other in small groups. It also shows that when teachers are explicit in the types of thinking they want their children to use, it encourages children to be more focused during the cooperative problem solving process.

Besides knowing how to solve problems using concepts, skills and problem-solving strategies, one's ability to control and monitor the thought processes is essential. Schoenfeld (1985) refers to these non-cognitive aspects of problem solving as metacognition. The role of metacognition within the heuristic framework of mathematical problem solving in small group setting was examined by Artzt and Armour (1992). They found that students went back and forth using different heuristics intermittently throughout the problem solving session and the attitudes of the high-ability group members manifested themselves in the subsequent behaviours of the group members.

In his review, Lester (1994) called for a shift in problem solving research from studying individual problem solving to problem-solving instructions in the classroom. He pointed out three key issues to be addressed seriously by researchers: '*The role of teacher in a problem-centered classroom*', 'W*hat actually takes place in problem-centered classrooms?'*, and '*Research should focus on groups and whole classes rather than individuals'*. This article described a programme that attempt to explore these issues.

Research Questions

The aim of this study was to find out how the use of cooperative problem solving strategies can improve the mathematical performance of students in applying more problem-solving strategies to solve non-routine problems. The second aim was to identify the metacognitive and affective behaviours manifested by the students during cooperative problem-solving carried out in the above study. The following questions were thus formulated:

1. Does the use of cooperative learning strategies improve the mathematical problem-solving performance of students?

2. Does the use of cooperative learning strategies result in students using more effective problem-solving strategies?

Research Design and Methodology

Sample and Grouping of Students

A total of 40 students from an all-girls secondary school took part in the study. The students came from a secondary three class in the Express stream. The sample was selected because the teacher (the teacher was the investigator and author in the study) was teaching them at the time of this study. The class consisted of students whose mathematics scores for their Secondary 2 Final Term Examination results were 63% and below, hence they were considered to be low achievers. For the purpose of this study, the students in the class were ranked according to their Secondary 3 Mid Year Mathematics Examination scores and then put into groups of 4, following the guideline from Slavin (1991) for assigning students to teams. Each group consisted of students from mixed ability. There were a total of 10 groups. The group, which consisted of the two top scorers in mathematics and the two lowest scorers in mathematics, was selected to examine the metacognitive and affective aspects of group problem solving. The investigator was interested to examine the interaction between the high and low achievers in this group.

Treatment

Design of Cooperative Mathematical Problem Solving (CMPS) Programme: The teacher designed a programme to be carried out in the classroom to teach mathematical problem solving. The design consisted of nine separate lessons, three of which were discussion sessions and the remaining six were cooperative problem solving sessions. In each lesson, a non-routine problem was used as a springboard to discuss specific problem-solving strategies, either with the teacher as the facilitator (in Discussion sessions) or in group problem solving (in Cooperative Problem Solving sessions). The discussion and cooperative problem solving sessions will be described further in the next section. The type of activities and key strategies in the CMPS Programme are summarized in Appendix 1.

<u>(a) The Discussion Sessions</u>
The aim of the discussion sessions was to help students acquire some useful problem-solving strategies in solving non-routine problems. The teacher used questions to prompt the students throughout the sessions.

The lesson consisted of eight components: *Key Strategies*, *Problem*, *Understand*, *Suggest a Plan*, *Implement*, *Verify*, *Conclusion and Possible Solutions*. '*Key Strategies and Problem*' showed the key strategies and the problem to be covered in the lesson.

In the '*Understand*' stage, the teacher posed questions and guided students towards understanding of the problem. In '*Suggest a Plan*', the students were prompted to suggest ways to tackle the problem. Next, the students were given time to implement their plans in the '*Implement*' stage. After implementation, the students were asked to check and verify their solutions. For lesson closure, a summary of key strategies and learning points were discussed.

The teacher role-modeled the problem-solving process by going through the problem with the class. She constantly posed questions to guide the students in the various stages of problem solving. Examples of questions were:

What is the question in the problem?
Can someone suggest a plan to solve this problem?
What is your plan? Explain the choice of your plan.
Paraphrase or explain the problems in your own words.
Does the answer make sense?
Did you have other methods to solve the problem?

<u>(b) The Cooperative Problem Solving Sessions and Roles of Teacher</u>

There were seven components in the cooperative problem solving sessions: *Key Strategies, Problem, Assign Roles to Students, Getting Started, Cooperative Problem Solving, Discussion of Solutions and Possible Solutions*. In '*Key Strategies and Problem*', the teacher and students considered the key strategies and the problem to be covered in the lesson. The problems used were also the "non-routine" or "non-standard" type, similar to those used in the Discussion sessions.

In '*Role Assignment*', the teacher highlighted and stressed the importance of active participation during the group discussion so that all of them could benefit from the session. She explained the roles of all the members in each group, namely: chairman, explainer, encourager and recorder. The chairman would ensure smooth discussion in the group and that no one should dominate the group. The explainer would explain when someone was not clear about any point. The encourager would encourage and praise members for their contributions. The recorder would record group solutions or any written explanation. The teacher emphasised that students should not just play the roles assigned to them, they also needed to be actively involved in the discussions. For the first session, the teacher gave some specific examples to show how the students should behave in each role. For example:

Chairman can say: "Wendy, do you have any ideas to add on?" or "Susan, what do you think of this plan?"
Encourager can say: "Wendy, you have given a good suggestion!" or "Susan, I think your method will work, but it can be very tedious to count all the lines (Mystic Rose Problem)."

Each student was assigned a role for every session and it was rotated so that at the end of the programme, each student had tried out all the different roles.

In '*Getting Started*', the teacher reviewed the key strategies covered in the last session which the students might find useful for the cooperative problem solving. She also reminded the students to organise into their groups with minimal noise and movement. In 'Cooperative Problem Solving', the students solved the problem in their groups and the investigator acted as facilitator. Generally, the teacher encountered three situations in the groups. Some groups were making very slow progress while others none at all. There were groups who were able to obtain a solution to the problem in a short period of time. The teacher provided assistance only when needed, posed questions to keep the group on track and provided encouragement as the group progressed through the problem solving. Examples of questions were:

Chairman, tell me what have your group done?
Explainer, can you explain why have your group decided to choose this plan?
Encourager, have all your group members understood the solutions?
Recorder, what is that you are writing down? Can you explain?
Did you use strategies learnt in the last session?

For groups which were not making progress or had difficulty tackling the problem, the teacher would guide them using open-ended questions to re-direct them to think about their plans and strategies. Examples of questions were:

Do you think you understand the problem completely?
Explainer, please paraphrase or explain the problem in your own words.
Chairman, tell me why your group find it difficult to continue with the problem.
What do you think might have gone wrong?

For groups which were ahead of time and had found a solution, the teacher would ask them to explain their solutions to her. When appropriate, she encouraged them to come up with alternative solutions, to generalize the problem to another situation or to extend the original problem to make it more challenging and interesting. Examples of questions were:

Chairman, what is the strategy your group has used?
Can you find another method to solve the problem?

Explainer, how can you explain your solutions to the class so that they can understand your group solutions?

Can your group modify the problem to make it more interesting and challenging?

When the teacher was satisfied that all the groups were making progress and all the groups had discussed the problem substantially, she would ask explainers from some groups to share their group solution with the class. When time permitted, the other groups would comment on their solutions, particularly useful strategies and their applications.

The aim of cooperative problem solving sessions was to provide opportunities for students to solve problems together in a group. Through the group interaction, students discussed strategies, explored problem solving and learnt from one another. While discussion sessions focused on class discussion with the teacher as the facilitator, cooperative problem solving sessions involved minimal guidance.

(c) The Choice of Problem Tasks

The problems selected for the programme were referred to as "non-routine" or "non-standard" and are usually not solved by simple recall or the application of familiar and standard algorithms. They were mostly problems adapted from the Shell Centre for Mathematical Education (1984) and from the book by Charles and Lester (1982). Such tasks require students to apply their thinking skills and problem-solving strategies in order to solve the problems. An example of problem used in the Discussion session is the **Tournament** Problem:

A tournament is being arranged. 22 teams have entered. The competition will be on a league basis, where every team will play all other teams twice – once at home and once away. The organizer wants to know how many matches will be involved.

Another example of problem used in the Cooperative Problem session is the **Mystic Rose** Problem:

The Mystic Rose has been made by connecting all 18 points on a circle to each other with straight lines. Every point is connected to every other point. How many straight lines are there?

Instrumentation

Data were gathered using two instruments. The first instrument was a problem solving achievement pre-test and post-test designed by the investigator for the purpose of this study. For comparison purposes, the problems selected were similar in terms of strategies used to solve them although they differed in their context. The

Friendship Bands and *Presents* Problems use strategies such as trying simple cases and looking for patterns. The *Number Assignment* and *Coding* Problems were designed to elicit strategies such as systematic listing and use of Decision Trees. The *Detective* and *Couples* Problems looked at the organisation of problems using a Matrix Table. Table 1 shows the problems used in the pre-test and post-test.

Table 1
Problems Used in the Pre-test and Post-test

Pre-test	**Post-test**
Friendship Bands A class chairman, Lisa, believed that making friendship bands could be one way to strength class spirit and friendship. To give it a try, she decided that each student would make a friendship band for every student in the class. How many friendship bands would have been made if there were 40 students in the class?	*Presents* At a Christmas party, each guest exchanged presents with every other guest. How many presents were exchanged if a total of 100 people were present at the party?
Number Assignment Peishan was to assign a number to each student in her class. She could only use the digits: 2, 5 and 6. Each student's name must have 3 digits. Can Peishan find enough numbers for all the 25 students in the class if repeated digits are used?	*Coding* A librarian, Miss Lee needs to do coding for some new library books using 3-digits number code. She realised that she only has number stickers for digit 4, 7 and 9 in her drawer. Are the stickers enough to make coding for 25 new books if repeated digits are allowed?
Detective A security guard at a bank caught a robber. The robber, the teller, and a witness were arguing when the police arrived. This was what the police learnt in the confusion. a. The names of the 3 men were Ali, Ben and Carey. b. Ali was the oldest of the three. c. The teller and Ben had been friends for many years. d. Ali was the brother-in-law of the witness. e. Carey graduated from school 5 years earlier than the robber. Name the robber, teller and the witness.	*Couples* Four married couples went to a baseball game the week before. The wives' names are Carol, Sue, Joanne and Ann. The husbands' names are Danny, Bob, Gary and Frank. Bob and Joanne are brother and sister. Joanne and Frank were once engaged, but broke up when Joanne met her husband. Ann has a brother and a sister, but her husband is an only child. Carol is married to Gary. For the other couples, who is married to whom?

The second instrument was a behavourial checklist to code and categorise behaviours of a group of students solving a problem together. The checklist, developed by the investigator based on ideas from Artzt and Armour (1992) and Foong (1990), explicitly described the metacognitive and affective behaviours captured within small-group problem solving. The checklist and examples of these metacognitive and affective behaviours obtained from the protocols were illustrated in Table 2.

Table 2
Cooperative Problem Solving Behavioural Checklist

Metacognitive Behaviours		**Examples of behaviours**
M1	Ask for clarification	a. "…the next question is 12 cubes high, so it means the centre one is 12, is it?" b. "…what do you mean by 3 ways?..." c. "How you calculate?"
M2	Remind problem requirement	a. "…wait, we see what they want us to do…" b. "…they want us to describe."
M3	Give suggestion	a. "Make a table." b. "Since the diagram is so big, you can count." c. "If we do the tree, easier right?" [Student suggested using Decision Tree because she thought it was easier.]
M4	Evaluate exploration	a. "It's not correct!" b. "Doesn't seem reasonable, does it?"
M5	Self-questions	a. "Find pattern already or not?" [Translate as: "Have we found the pattern or not?"] b. "Describe what? All describe here already." c. "What am I doing?"
M6	Revise plan	"…so find another less complicated pattern…"
M7	Check computation	a. "127, correct." b. "2n – 1 will be …, 4 – 1 is 3, 3×2 is 6, 6 – 1 is 5, 5×3 is 15. Okay correct."
M8	Explain solution	a. "We are trying to get a general rule so that we can substitute any number inside." b. "…two 7, two 35, two 1, so it's all zeros."
Positive Affective Behaviours		
P1	Persist in task	a. "Finish this two…finish this…" b. "…n(n-1)? Okay, I try."
P2	Encourage	"…okay, just voice out your objection…"
P3	Praise	a. "So smart!" b. "Simplify expert!" c. "Ha, you become expert already!"

Negative Affective Behaviours		
N1	Self-evaluation	a. “I don’t understand the question.” b. “Mine one is rubbish one!” [Student commented that her method was wrong.]
N2	Complaint	a. “I hate drawing this line!” b. Long winded you! This is call long winded!” c. “Aiyah! I don’t understand what we are trying to find!” d. “Stupid, got so many ways to go.”
N3	Emotions	a. “Aa…” [groans] b. “Aiyah!” c. “Aiyah, why must do this!"

Data Collection and Analysis

The pre-test and post-test were administered to the class of 40 students before and at the end of the CMPS programme to investigate if there were any significant differences in the overall performance, using paired T-test at 0.01 level of significance. These tests were further analysed to identify the problem-solving strategies used by the students.

The data obtained from the pre-test and post-test were analysed to study the four sub-components of problem solving as measured by the tests. A holistic analytic marking scheme, adapted from Charles, Lester and O’Daffer (1987), was used to score the pre-test and post-test because it reflects the different stages of problem solving and not just the final answers. The four sub-components of problem-solving were: Understand, Plan, Operations and Answers. Table 3 (see next page) shows the holistic analytic marking scheme and the criterion for awarding marks in each stage.

The problem-solving strategies used by the students to solve all the problems in the pre-test and post-test were further identified to compare the problem-solving strategies used by students before and after the CMPS programme. The investigator classified these strategies into 11 different strategies used for this study. They were:

1. *Try simple cases*
2. *Look for pattern*
3. *Find a general rule*
4. *Make a table*
5. *Use a Matrix table*
6. *Use of a Decision Tree*

7. *Systematic listing*
8. *Guess and check*
9. *Logical reasoning*
10. *Unsystematic listing*
11. *Number manipulation*

Strategies 8 to 11 were pre-instructional strategies used by students to solve problems designed for this study. If the students did not show any strategies in their answers, it was classified as "*No method shown*". For students who did not complete or attempt some problems, their work was classified as "*No attempt*". Although the sample size was 40, the data for two students were incomplete and hence were not included in the analysis for Research Question 1 and 2. Samples of the students' work are shown in Appendix 2.

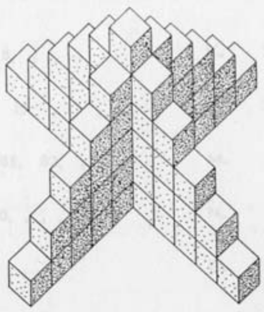
Table 3
Holistic Analytic Marking Scheme

No.	Stage
1	**Showing your understanding of the problem** (max. of 2 marks) 2 – Complete understanding of the problem. 1 – Part of the problem misunderstood or misinterpreted. 0 – Complete misunderstanding of the problem.
2	**Showing your plan** (max. of 3 marks) 3 – Plan could lead to a correct solution if implemented properly. 2 – Partially correct plan based on correct interpretation of part of the problem. 1 – Attempt to show some plan based on interpretation of part of the problem. 0 – No attempt, or totally inappropriate plan.
3	**Showing all your steps and operations** (max. of 3 marks) 3 – Complete steps and operations that lead to correct answer. 2 – Incorrect answer due to copying error, computational error although answer follows logically from a correct plan. 1 – Incorrect answer although attempt to show some logical steps and operations that follows logically from an inappropriate plan. 0 – No answer, or wrong answer based on an incorrect plan.
4	**Showing your correct answer in complete sentence** (max. of 2 marks) 2 – Correct answer and complete sentence for answer. 1 – Correct answer but no complete sentence for answer. 0 – Incorrect answer.

To study the behaviours of students in cooperative problem solving, a group of four students from the sample was selected. Verbal data was collected on their behaviours in the form verbatim transcripts of the recorded tapes. Only one group of students was selected so that the investigator could do a detailed qualitative analysis of the four students by following them throughout the whole programme. The group was audio-taped during two Cooperative Problem Solving sessions working on two problems, *Skeleton Tower* and *Transport* Problem.

<u>*Skeleton Tower* Problem:</u>

Skeleton Tower



(i) How many cubes are needed to build this tower?

(ii) How many cubes are needed to build a tower like this, but 12 cubes high?

(iii) Explain how you worked out your answer to part (ii).

(iv) How would you calculate the number of cubes needed for a tower ***n*** cubes high?

<u>*Transport* Problem:</u>

Every morning, Liling can take a bus, MRT (Mass Rapid Transit) or walk to school. After school, she can take a bus or MRT home. Sometimes, her father fetches her home or she gets a lift from Yanling who lives near her.

(a) *Describe 3 ways how Liling can travel to and from school.*

(b) *What is the total number of possible ways Liling can travel to and from school? Explain your answers.*

The verbal data collected was analysed using the behavioural checklist described earlier. Inter-coder agreement of the behavioural checklist was established by asking another mathematics teacher to code the behaviours based on her perception of the definitions given in the checklist. Table 4 shows the results of the analysis of inter-coder agreement.

Table 4
Results of Analysis of Inter-coder Agreement

Behaviours	Proportion Agreement	Percentage Agreement
Metacognitive	15 out of 19	78.9%
Positive Affective	6 out of 6	100.0%
Negative Affective	8 out of 9	88.9%

Generally, the result was satisfactory. The two coders negotiated agreement on the ones where there was disparity. Next, the investigator proceeded with the coding of all the protocols used in the study. The checklist was completed for the protocol analysis to give a quantitative measure of behavioural frequencies within the group.

Findings, Analysis and Discussions

In the following section, the results of this study were analysed and discussed in sequence of the research questions.

Research Question 1

Results from the analysis of students' problem-solving performance using the holistic analytic marking scheme showed that the students scored significantly better in the post-test as compared to the pre-test. They performed better in all the four components identified in the problem-solving process: Understand, Plan, Operation and Answer. Equipped with more problem-solving strategies after the CMPS programme, the students showed better understanding of non-routine problems, hence they were able to come out with appropriate plans and then proceed with the operations to solve problems successfully in the post-test. They were more organised in their operations and were able to apply more effective and more specific problem-solving strategies leading to the correct solutions. All the standard deviations of the results for the post-test were smaller as compared to those for the pre-test, indicating that the spread of students' scores was less apparent in the post-test. This suggests that the students generally had improved in their problem-solving performance after going through the cooperative problem solving programme. Table 5 shows the results for the pre-test and post-test.

Table 5
Comparison of Means and Standard Deviations in Pre-test and Post-test

PERFORMANCE	PRE-TEST		POST-TEST		T-VALUE
	Mean	Standard Deviation	Mean	Standard Deviation	
Understand component	7.05	1.18	7.82	0.56	3.67*
Plan component	9.68	1.85	11.66	0.91	6.84*
Operation component	8.45	2.32	11.13	1.23	6.69*
Answer component	5.53	1.90	7.37	1.05	5.99*
Overall	30.71	6.74	37.97	3.28	6.57*

$*p<0.01$

Research Question 2

The results showed that the students applied problem-solving strategies such as try simple cases, look for patterns and find a general rule to simplify problems. They were able to organise problems using diagrams such as drawing tables, Matrix tables and Decision trees. This finding suggests that the CMPS programme was effective in enabling students to use the problem-solving strategies covered in the programme. The students were able to apply the two main problem-solving heuristics taught, namely simplifying problems and organizing systematically. Further, the use of pre-instructional strategies such as guess-and-check, unsystematic listing and number manipulation was less frequent in the post-test. The common use of logical reasoning also declined as the students were able to apply some specific strategies to solve problems. Table 6 (see next page) shows the results of the comparison of problem-solving strategies used by the students for the pre-test and post-test.

Besides acquiring specific problem-solving strategies, the study identified metaccognitive and affective behaviours manifested in four students when they solved two problems together. These behaviours have exerted positive influence on their attitude towards problem solving as well as enhancing their problem-solving abilities. Table 7 (see page 73) shows the protocol analysis summary of the *Skeleton Tower* and *Transport* Problem. As seen from the table, 'asking for clarification' accounted for 40.4% of the metacognitive behaviours identified from the group of students under study. Besides asking for clarification, the group environment enables the students to explain their solution (9.6%), check one another's computation (12.8%) and evaluate the progress of their exploration (8.5%). Due to the non-routine nature of the problems, negative affective behaviours were detected in the group.

Table 6
Comparison of Problem-Solving Strategies Used by Students in Pre- and Post-test

Problem-solving Strategies	(Pre-test) Friendship Bands	(Post-test) Presents	(Pre-test) Number Assignment	(Post-test) Coding	(Pre-test) Detective	(Post-test) Couples
Try simple cases	0	26				
Look for patterns	1	30				
Find a general formula	0	23	0	8		
Make a table	0	26			2	28
Use of Matrix table			0	12		
Use of Decision tree			16	12		
Pre-Instructional Strategies	(Pre-test) Friendship Bands	(Post-test) Presents	(Pre-test) Number Assignment	(Post-test) Coding	(Pre-test) Detective	(Post-test) Couples
Guess-and-check					3	0
Logical reasoning	30	10	2	0	24	9
Unsystematic listing			11	8		
Number manipulation	1	0	1	0		
	(Pre-test) Friendship Bands	(Post-test) Presents	(Pre-test) Number Assignment	(Post-test) Coding	(Pre-test) Detective	(Post-test) Couples
No method shown	6	0	6	0	9	0
No attempt			2	0		

Occasionally, the students expressed negative emotions (40%) and complained (50%) about the problems. Nevertheless, the group was able to persist (50%) and solve the two problems. Positive affective behaviours such as ‘encourage’ (12.5%) and ‘praise’ (37.5%) were identified in the study.

Table 7
Frequency of Metacognitive and Affective Behaviours Shown by Group Solving ***Skeleton Tower*** *and* ***Transport*** *Problem*

	Skeleton Tower Problem	**Transport Problem**	**Total**	**%**
Metacognitive Behaviours				
M1 – Ask for clarification	14	24	38	40.4
M2 – Remind problem requirement	3	6	9	9.6
M3 – Give suggestion	4	5	9	9.6
M4 – Evaluate exploration	1	7	8	8.5
M5 – Self-questions	2	5	7	7.4
M6 – Revise plan	1	1	2	2.1
M7 – Check computation	10	2	12	12.8
M8 – Explain solution	6	3	9	9.6
			94	**100%**
Positive Affective Behaviours				
P1 – Persist in task	3	1	4	50.0
P2 – Encourage	0	1	1	12.5
P3 - Praise	3	0	3	37.5
			8	**100%**
Negative Affective Behaviours				
N1 – Self-evaluation	1	0	1	10
N2 – Complain	2	3	5	50
N3 - Emotions	4	0	4	40
			10	**100%**

Implications for Teaching

Results from this study seemed to suggest that the use of small group cooperative problem solving can be an alternative instructional strategy for organising and teaching mathematical problem solving in the classroom. Teachers can design lessons involving group work to equip students with various problem-solving strategies, skills and heuristics so that they may become better problem-solvers. After being exposed to a variety of problem-solving strategies, the students became more confident in applying these strategies, as seen in the more frequent use of more effective problem-solving strategies in the post-test compared to the use of less potent strategies such as guess-and-check, unsystematic listing or number manipulation.

Further, results obtained from the protocol analysis indicate that students could learn to engage in more metacognitive behaviours such as *asking for clarification, remind problem requirement, giving suggestion, checking and explaining.* We have observed that the cooperative problem solving environment provides opportunity for the students to ask questions, raise doubts, seek clarification and offer explanations to one another in the group. It is hoped that such engagement and participation during cooperative learning sessions could help students improve in their problem-solving ability. Overall, these meta-cognitive behaviours exhibited by the students may be emphasised or modeled by teachers during whole-class discussions so that students become conscious of those behaviours. This, in turn, can be internalised in the students as they learn to work on problems individually. Teachers can also reinforce positive affective behaviours and discourage negative affective behaviours in students to help them in the problem solving processes. Here, the teachers can model behaviours such as praising and encouraging students to persist in their tasks, reminding them to approach the problems positively, rather than indulge in excessive emotions or complaints whenever they encounter difficulties.

Limitations and Recommendations for Further Research

There are some limitations to this study such as the sample which was all girls, hence the results obtained cannot be generalised to the boys. The problems selected for the study could not be assumed to be representative of all typical problems in the mathematical problem solving context. They were selected based on their relevance and suitability to the aims of the programme under study. Also owing to the small number of problems and subjects involved, no attempt was made to generalise the problem solving processes and behaviours beyond this research. The use of protocol analysis of student behaviours can only record the verbalistion of students solving problems. Hence any mathematical covert thoughts and processes of the students would be impossible to capture. The coding of the transcripts could be subjective depending on the interpretation and perception of the coders.

Nevertheless, the study was intended to ignite more interest in the area of teaching mathematical problem solving using cooperative learning. Like all exploratory studies, this study has also raised a few questions for further research. Firstly, there is certainly a need for another study to apply the same cooperative learning programme to more diverse groups of sample to see if the programme is as effective and successful as it was in this study. For example, this programme can be tested with boys or students in co-educational schools, students in the Normal (Academic), Normal (Technical) and Gifted Stream. This can be a very useful area of research because different schools will require appropriate and suitable programme to cater for the needs of their students. Also, the student-teacher interaction during the discussion and cooperative problem solving sessions could be further studied to examine how the teacher's role as a facilitator enhances students' heuristics, metacognitive and social skills. Davidson (1990) believed that the teacher's role in structuring learning situations cooperatively involves clearly specifying the objectives for the lesson, placing students in learning groups and providing appropriate materials, clearly explaining goal structure and learning task, monitoring students as they work, and evaluating students' performance.

Concluding Remarks

In view of the importance of teaching mathematical problem solving in schools, research in this area is of utmost help to mathematics teachers and educators. This study has demonstrated that the use of cooperative learning environment had a positive effect on the acquisition of problem-solving strategies. The students engaged in a cooperative problem-solving process improved in their mathematical problem-solving performance. Besides acquiring more effective problem-solving strategies, the students exhibited metacognitive behaviours that help them to be become more effective problem solvers. This study suggests an alternative to both traditional whole class expository instruction and individual instruction in teaching mathematical problem solving in the classroom.

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Appendix 1

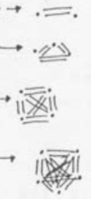
Outline of Cooperative Mathematical Problem Solving (CMPS) Programme

Lesson	Type of Activity	(*Heuristics**) Key Strategies	Problem
		PRE - TEST	
Session 1	Discussions	***Organise systematically**** ***Simplify problem*** ***Draw a helpful diagram*** Try some simple cases Make a table Look for patterns Use the patterns Find a general rule	*Tournament*
Session 2	Cooperative Problem Solving	***Organise systematically*** ***Draw a helpful diagram*** Try some simple cases Make a table Look for patterns Use the patterns Find a general rule	*Mystic Rose*
Session 3	Cooperative Problem Solving	***Organise systematically*** ***Simplify problem*** Look for patterns Use the patterns	*Skeleton Tower*
Session 4	Cooperative Problem Solving	Find Number Patterns	*Finding Number Patterns*
Session 5	Cooperative Problem Solving	Find number patterns Use the patterns Find a general rule	*Numbers in Triangle*
Session 6	Discussions	***Organise systematically*** ***Draw a helpful diagram*** Matrix table Elimination	*Cars*
Session 7	Cooperative Problem Solving	***Organise systematically*** ***Draw a helpful diagram*** Matrix table Elimination	*Contest*
Session 8	Discussions	Systematic listing Look for pattern Use the patterns	*Combinations*
Session 9	Cooperative Problem Solving	Systematic listing Decision Tree	*Transport*
		POST - TEST	

Appendix 2a

Scoring of Present Problem using Holistic Analytic Marking Scheme - Sample 1

2guests →
3 guests →
4 guests →
5 guests →



2 presents
6 presents
12 presents
20 presents

No. of Guests	No. of Presents Exchanged
2 × 3	2 → 6
3 × 4	6 → 12
4 × 5	12 → 20
49 × 50	2,450

2,450 presents were exchanged if a total of 50 people were present at the party.

Understand **2 marks**	Complete understanding of the problem.	**Score** **2**
Plan **3 marks**	Correct plan used. The student simplified the problem, tried simple cases and then looked for a pattern.	**3**
Operation **3 marks**	Complete steps and operations that followed from the correct plan, hence leading to the correct answer.	**3**
Answer **2 marks**	Correct answer with complete sentence given.	**2**
Total **10 marks**	**Score**	**10**

Problem-solving strategies identified: Try simple cases, Make a table

Appendix 2b

Scoring of Number Assignment Problem using Holistic Analytic Marking Scheme
- Sample 3

3 digits
Listing out all the numbers with digits 2, 5 & 6 in it.

256 562
526 652
625 265

I don't think Peishan will be able to find enough numbers.

Understand **2 marks**	Incomplete understanding of the problem as the student did not use repeated digits.	**Score** **1**
Plan **3 marks**	The student listed all the digits based on the incomplete understanding of the problem, hence missing out numbers with repeated digits.	**1**
Operation **3 marks**	The student followed an inappropriate plan and hence was only able to list out 6 numbers which was the incorrect answer.	**1**
Answer **2 marks**	Incorrect answer because there should be enough numbers for 25 students if repeated digits were allowed.	**0**
Total **10 marks**	**Score**	**3**

Problem-solving strategy identified: Unsystematic listing